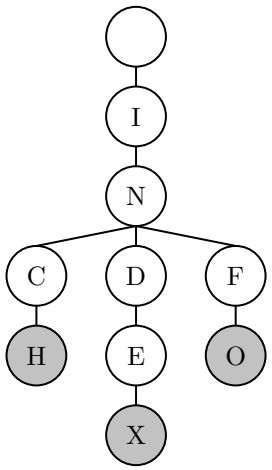
CS 61B Spring 2021

Tries and More Graphs

Discussion 11: April 5th, 2021

1 Trie Your Best

(a) What strings are stored in the trie below? Now insert the strings *indent, inches, and trie* into the trie.



(b) What is the runtime to find out if a given string is in the tree? What is the runtime to add a string to the tree? Describe your answers in terms of N, the number of words in the trie. You may assume the max length of any word in the trie is a constant.

(c) *Extra:* How could you modify a trie so that you can efficiently determine the number of words with a specific prefix in the trie? Describe the runtime of your solution.

2 A Tree Takes on Graphs

Your friend at Stanford has made some statements about graphs, but you believe they are all false. Provide counterexamples to each of the statements below:

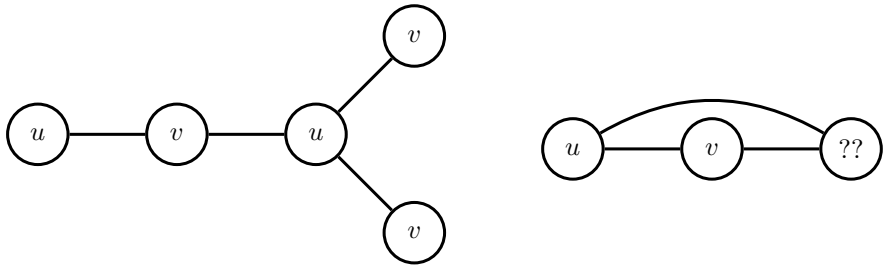
(a) "Every graph has one unique MST."

(b) "No matter what heuristic you use, A* search will always find the correct shortest path."

(c) "If you add a constant factor to each edge in a graph, Dijkstra's algorithm will return the same shortest paths tree."

3 Graph Algorithm Design

(a) An undirected graph is said to be bipartite if all of its vertices can be divided into two disjoint sets U and V such that every edge connects an item in U to an item in V. For example below, the graph on the left is bipartite, whereas on the graph on the right is not. Provide an algorithm which determines whether or not a graph is bipartite. What is the runtime of your algorithm?
Hint: Can you modify an algorithm we already know?



(b) Consider the following implementation of DFS, which contains a crucial error:

```
create the fringe, which is an empty Stack
push the start vertex onto the fringe and mark it
while the fringe is not empty:
    pop a vertex off the fringe and visit it
    for each neighbor of the vertex:
        if neighbor not marked:
            push neighbor onto the fringe
            mark neighbor
```

First, identify the bug in this implementation. Then, give an example of a graph where this algorithm may not traverse in DFS order.
Hint: When should we be marking vertices?

(c) *Extra:* Provide an algorithm that finds the shortest cycle (in terms of the number of edges used) in a directed graph in $O(EV)$ time and $O(E)$ space, assuming $E > V$.